You may write in this booklet, but make sure to copy all answers onto the answer sheet as that will be the only paper collected.

1. $2025 = 1^3 + 2^3 + 3^3 + ... + n^3$. What is n?

2. $2025 = a^2 + b^2$. What is $a + b$, given that a and b are natural numbers?

3. Compute the remainder when $9 + 9^2 + 9^3 + ... + 9^{2025}$ is divided by 13.

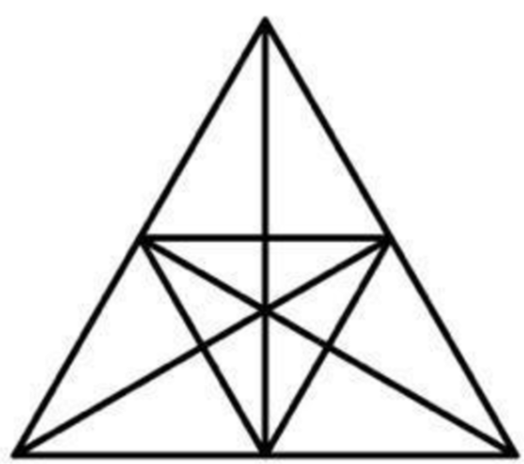
4. Suppose x and y are real numbers such that
$$\frac{x-y}{x+y} = \frac{2024}{2025}$$
What is $\frac{x}{y}$?

5. Let f be a function with the following properties:
   (i) $f(1) = 1$
   (ii) $f(2n) = n \cdot f(n)$ for any positive integer n.

   What is the value of $f(2^{10})$?

6. Tim flips a coin five times and heads occur three times. Given this information, what is the probability that the first two flips were heads?

7. How many triangles are in the following image?



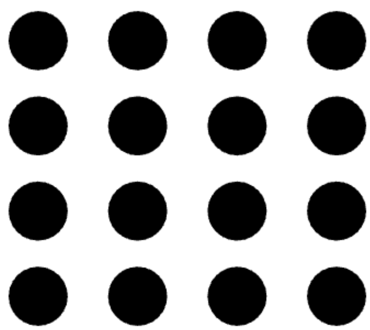
8. Define the operation, ❖, as

❖$(a, b, c) = ab - c.$

What is ❖(❖$(1, 2, 3)$,❖$(2, 3, 1)$,❖$(3, 1, 2)$)?

9. A cyclist, Dr. Hughes, is stationary at a point A on the Northwest River Trail, a straight trail, when he hears the horn of a car coming from behind. After 17 seconds the car passes by him. It is known that the car was 1200 meters away from the cyclist when it honked, and it was travelling at a constant speed. Supposing the speed of sound is 400 meters per second, find the speed of the car.

10. A set of four points is randomly chosen from the grid shown below. Each four point set has the same probability of being chosen. What is the probability that the points lie on the same straight line?



11. The digits 1, 2, 3, 4, 5, and 6 are arranged in some order to form a six-digit number with these properties:

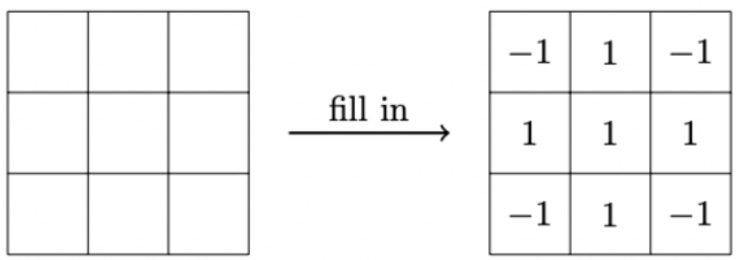
    (i) The six-digit number is divisible by 6.
    (ii) The number formed by the five left-most digits is divisible by 5.
    (iii) The number formed by the four left-most digits is divisible by 4.
    (iv) The number formed by the three left-most digits is divisible by 3.
    (v) The number formed by the two left-most digits is divisible by 2.
    (vi) The left-most digit is divisible by 1.

    There are two such numbers. Compute their sum.

12. What is the last time in the day that the hour and minute hands of a standard clock overlap?

13. Alice and Bill live at opposite ends of the same street. They leave their houses at the same time and each walk, at constant speed, from their house to the other house and back. The first time they meet, they are 400 yards from Alice's house, and the second time they meet, they are 300 yards from Bill's house. Both times they are traveling in opposite directions. What is the distance, in yards, between the two houses?

14. In each cell of the 3×3 table on the left below, fill in 1 or -1, such that the product of numbers in each row or column is equal to 1 as shown in the example on the right. How many ways are there to fill in this table?

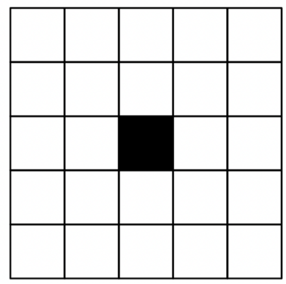


15. What is the fifth smallest positive integer such that if the leftmost digit is removed, the resulting number is one-fifth of the original?

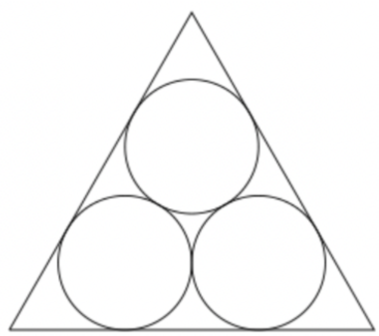
16. A sequence is generated using this procedure. The first number in the sequence is 1. To generate each successive term, a fair coin is flipped. If the result is heads, the next term is found by first multiplying the previous term by 2, and then subtracting 1. If the result is tails, the next term is found by first multiplying the previous term by 3, and then subtracting 1. What is the probability that the fourth term in the sequence is even?

17. If $f(x) = x^2 + 1$, what is the value of $f(f(f(f(0))))$?

18. The 5×5 grid shown below contains a collection of squares with sizes from 1×1 to 5×5. How many of these squares contain the black center square?



19. In the diagram below, three circles of equal size are inscribed in an equilateral triangle, so that they are tangent to each other and to the indicated sides of the triangle.



If each circle has radius 1, what is the side length of the triangle?

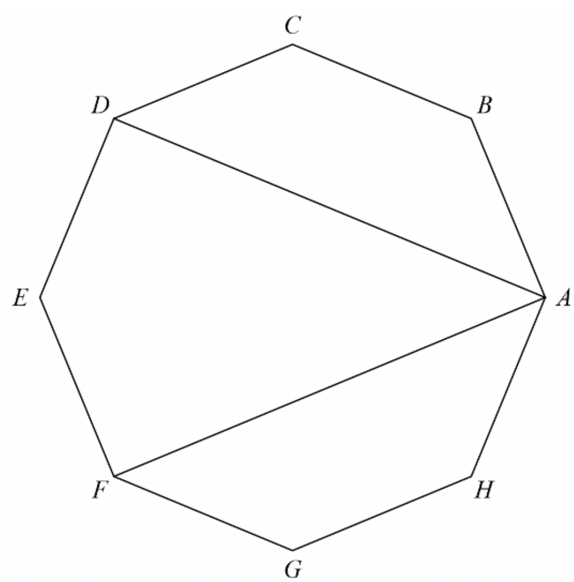
20. Let a function $f(x)$ satisfy the following condition:

$$f(x+1) = \frac{2}{1 + \frac{1}{f(x)}}, \text{ for any } x \geq 0$$

If $f(10) = \frac{2048}{2051}$, then what is $f(0)$?

21. A grasshopper starts at the 12 on a large 12-hour clock. She jumps to 1, then jumps to 2, and continues this way, returning to the 12 after 12 jumps, then jumping to the 1 on the 13th jump, and so on. After 2025 jumps, what number is the grasshopper on?

22. In the regular octagon $ABCDEFGH$, what is the number of degrees in the measure of angle $\angle DAF$?



23. Compute the perimeter of the triangle formed by the line $3x + 4y = 1$ and the x- and y-axes.

24. The number **210** is the product of two consecutive positive integers and is also the product of three consecutive integers. What is the sum of those five integers?

25. Suppose x and y are real numbers such that

$$x - \frac{1}{x} = y + \frac{1}{y}$$

What is the value of $x^2 + \dfrac{1}{x^2} - (y^2 + \dfrac{1}{y^2})$?

26. How many positive integer divisors does 2025 have?

27. If the 13 letters in ELIZABETHTOWN are arranged at random, then what is the probability that TIE will be spelled out anywhere in the arrangement?

28. Each person in a ticket line gets either one or two tickets. You are the 51st person in line. Forty percent of the people in front of you get one ticket each, and the rest of the people in front of you get two tickets each. How many tickets will have been given out when you reach the ticket booth?

29. What is the value of the digit K that will make the number $481,5K6$ divisible by 2, 3, 4 and 9?

30. If $\cos\theta = 0.1$, then what is $\cos 3\theta$? ***(Remember: Show all work and provide justification for this problem on the back of the answer sheet.)***